Connection Theory

Abstract: TBD

Contents

Adam In Tae Gerard **Rev. 0.69** **03.12.2020**

Affiliation(s): *Professional, Academic*

CONNECTION THEORY

Adam In Tae Gerard

Affiliation(s): *Professional, Academic*

Abstract: TBD

1. Introduction

The aim of this paper began as an inquiry into two connected strands of thought and debate in philosophy and gradually coalesced into a singular line of thought supporting a way of thinking about basic mathematical entities.

In my opinion, the most fertile areas of mathematical research either began in paradox or in the cultivation of a new theory of robust mathematical entities (sets, functions). Of these, sets are well-understood due to the number of results obtained using ZFC set theory and the wide variety of alternative axiom systems that exist that explore ways that sets can be conceived, constructed, and ordered.

Functions, however, remain largely opaque except for circumstances in which functions are analyzed according to some background set-theory (e.g. functions understood through set-builder notation or through set-theoretic extensions or intensions).

2. Philosophical Discussion

Mathematics involves the close **interaction** between **representation systems** (language, measuring devices, models, etc.) and **patterns** (mathematical relationships). I will remain silent about the *exact* nature of that **interaction** (a traditional area of inquiry in the epistemology of mathematics) and **patterns** (the metaphysics of math).

I *will* say that the **interaction** is much more tightly knit - **representation systems** are defined by grammatical or procedural rules that delimit the kinds of patterns they can talk about.

Thus, **interaction** need not be understood as a claim about a kind of *Fregean correspondance*. It also doesn't collapse the view into *Formalism*. Specifically, I do believe that there are (real) patterns but they are *revealed* in and through language - that math is not *merely* procedural symbol manipulation but a genuine discovery that involves more than the application of existing axiom systems or their finite creation.

The real patterns need not be understood as Platonic Forms either. They could, for instance, subsist in language itself. On such a view, mathematical truths are not merely the manipulation of finite symbols (which is a far too restrictive view of language and logic - one that is almost exclusively proof-theoretic in scope ignoring ontological engineering and semantics).

In fact, Connection Theory would represent a novel way to formally understand and define the mathematical relationships *between* these linguistic systems. We think *that* is valuable in its own right. Representation systems expressed in a variety of languages and diagrammatic means throughout mathematics would be united and given the same treatment (analogously speaking) that Set Theoretic Algebraic structures presently do. Perhaps this concept would support a reified notion of structure (but lacking the metaphysically laden overtones of many of the early structuralist debates in the philosophy of mathematics).

There are several others who have already articulately defended this view[1]. I see it as a kind of neo-Pythagoreanism that remains silent about the appropriation of ancient metaphysical categories to organize taxonomies of mathematical phenomena. Rather, we access mathematical patterns through language and their extra-linguistic nature is obscured since the only substrate or medium in which we can interact with them is in language. This is closest to a kind of epistemic Kantianism with a close emphasis on the interaction between language and pattern as a nod to Wittgenstein.

It is Pythagorean because this way of viewing **interaction** blurs or eliminates the kinds of **abstract / concrete** distinction common to Platonic philosophies (and

[1] See Geoffrey Hellman's modal structuralism.

which have never, to my mind, been justified through a positive argument for their use in the first place – they are assumed, even by Plato and then employed to justify the larger philosophical tasks).

Mathematical theories can be of several kinds[2]:

1. *Existence* assertions.
2. *Satisfaction* conditions.

Mathematical theories that encompass a novel language are of the former (the ontological commitments of the underlying language lay a foundation for other dependent theories or results). Consider for a moment *ZFC set theory* and compare it to *Group Theory* expressed within set notation. The former describes a conception of sets and their elementary relationships (inclusion operator). The latter describes set-theoretic algebraic structures that fit a certain description (satisfy the axioms of *Group Theory*).

The Fundamentality Principle.

> If a mathematical theory T^* is a (complete) model for another theory T and T is *homomorphic* to, *isomorphic* to a fragment of, or is a *subset* of T^*, T^* is more fundamental than T.
>
> **Remark.** *ZFC Set Theory* is a model for the *Ordinals*. Thus, *ZFC Set Theory* is more fundamental than the *Ordinals*.

Objection: but isn't it the case that for any theory T, we may construct a theory T^*, such that T is a subset of T^*, and such that any model of T^*, is a model of T. Wouldn't that imply that then, by induction, that for every theory T there are an infinite number of more fundamental theories?

Reply: Two replies. First, yes. As counterintuitive as that might be, so it is that T^* is more fundamental than T. But, note that "fundamentality" here is not a loaded notion. It is not, for instance an ontological claim. Merely a claim about semantic and syntactic power (e.g. – expressivity). And, that should not be bewildering at all. It's standard practice.

---

[2]

While that's true, the objection does hit it's mark somewhat. What we're clearly going after is the concept of "foundational primacy" (or some equivalent) since we're doing foundations after all. And, the notion of "fundamentality" above is too broad for the intended work. We might add to the principle above, the following:

New Math Foundations:

1. For every mathematical theory T, there is a fundamental theory T^* such that every theory T is a subset of or isomorphic to a fragment of T^*.
2. As such, there a theory T^* that all other mathematics T can be expressed in.
3. And, T^* can be expressed in a more fundamental theory T^{**}.

Remark. Here, T^* would be analogous to ZFC Set Theory and Category Theory. T^{**} is Connection Theory (within which both ZFC Set Theory and Category Theory can be expressed).

Here, our objective is to define a kind of theory T^{**} sufficient for all known mathematical theories, to add to this body (at the foundational level), and to tighten up some of the common techniques and diagrams used throughout mathematics (but which themselves lack any rigorous justification or foundation).

Moreover, I shall assume that a language L that is human-readable (preferably a full Natural Human Language) that can be recovered in a language L^* that is:

1. At least partially human-unreadable.
2. Beyond the ontological categories undergirding all human language and taxonomic categories.

Is *Posthuman*.

Traditionally, there are three basic, fundamental, ontological categories that everything else has been understood to be one or more of:

Object / Thing / Entity / Individual / Substance - The reference of a noun word or name. A square, a cat, the paradigmatic philosophical atom of Democritus. The person the word "Democritus" (a proper noun) refers to. A monad in

Leibniz's system. The universe. Etc. Individual here usually means "specific item" rather than "person" (though "persons" are usually understood to "individuals" in both senses).

Leibniz’s *Principle of the Identity of Indiscernibles* and the concept of ‘Object’.

1. Robust – PII – Substantive notion of an ‘Object’ – e.g. – as an “individual”.
2. Weak (Ersatz) – Referent of singular term of symbol.

Property / **Attribute** / **Characteristic** / **Universals** - a feature or attribute of an object. Some people think that objects are just properties or that objects are a special kind of property (*haecceity*) into which other properties "stick into like a pin-cushion".

Relationships / **Relations** - A relationship is usually taken to be something that obtains between two or more objects. A cup rests on a table. Mary is the wife of Tom. 1 + 1 = 2.

Counter Argument / Reply: Connectors are not objects in the strong sense, though we may define a representation theorem in the weak sense.

If the claim is that “connectors are really (ersatz) objects (because we can specify a morphism between a diagram of a connector to a label like ‘A’ or ‘B’)”, the reply is straightforward. The relationalist retreats to the weaker claim, “yes, but we’ve reduced objects to connectors – they are ultimately what names refer to and connectors are just relations (or relational)”.

Connectors are *more than mere* relations (thus far so conceived). Consider two depictions using a sub-fragment of Connection Theory:

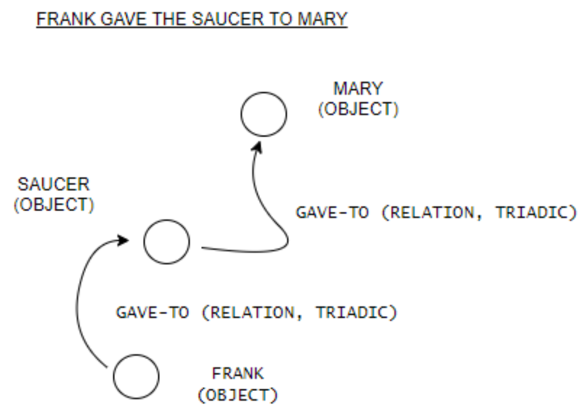


Fig. – Standard arity 3 relation.

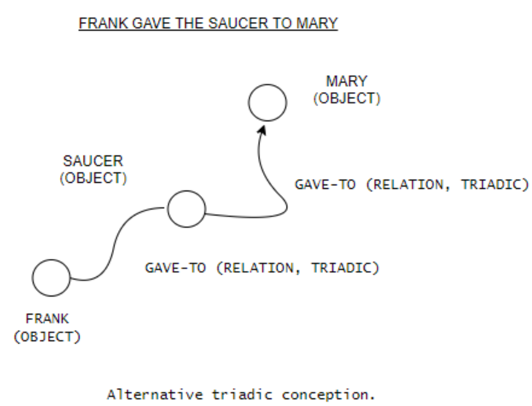


Fig. – Alternative depiction of a standard arity 3 relation.

Two depictions of the same relation are shown above. They are diagrammed as "*snakes*" (functions, morphisms, mappings, relations). Many-to-one or one-to-many relations are depicted as "*spiders*". These depictions are not specific to Connection Theory – these are standard ways to demonstrate the relationships between databases, etc. These ways of illustrating or representing relations are very simple – relations are extremely simple – and represent the conceptual limit of their illustration. Whether we substitute an arrow with some other equivalent symbol is irrelevant here. All relational diagrams and their possibilities are contained herein.

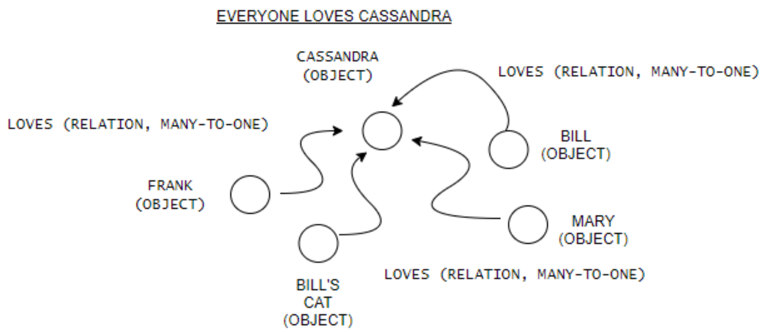


Fig. – Standard many-to-one relation.

Properties of relations and higher-order relations.

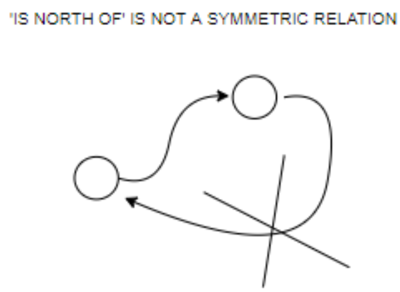


Fig. – Properties of relations – two "snakes" – second simplest "spider".

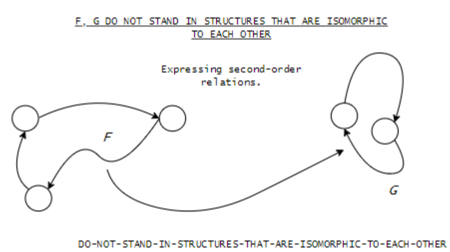


Fig. – Relations between relations – still a "snake".

To propel the theory to its wider objectives (to break free of existing language constraints) we will be forced to confront the basic components that make up all known human writing systems:

1. *Lines* (and *points* which have been so problematic in physics) of varying lengths in *Space*
2. *Space*
3. *Procedures/Processes*
4. *Curves* (from *lines* in *space*) and *Shapes*

Which form all *symbols* of any language whether pictographic/cuneiform or not, whether Romantic, Cyrillic, or Asiatic. As stated, these ontological categories have exerted tremendous practical limitations – the use of points (and frustrations with them) has led to String Theory in physics, mereology and point-free foundations in geometry, etc.

<u>All</u> human language is spatial whether body language, written language, symbolic, or auditory. We speak through space using sound to project meaning. We write on paper words (or record them onto machines through written software) to bind their meaning through time granting them at least a temporary reprieve from the momentary transience of sound.[3]

Can we break free from the language-based geometric limitations of two-dimensions? In doing so, will we see a great leap in human thinking (from a cognitive and language standpoint)? Can that standpoint alone lead us to higher thought-processes, ways of thinking about the world, and problem resolution (presumably both through better solution-formulation but also from seeing the world more clearly)?

---

[3] One of my goals early on as a poet was to construct 3-dimensional poems to illustrate the unity of the concepts of *subjectivity* and *objectivity* – that these were better understood geometrically. I think one or two poets eventually created poems that were strictly 3-dimensional. I had intended that my poems would be rotational, so that be changing one's literal vantage point, the text would appear differently. (Image a cube in three-space such that every face has a three-line poem written so that if you stood 45 degrees from one vantage point a new poem would appear – half of one face and half of another.)

While I never got around to that, the idea eventually revealed to me the limitations of known-human writing systems – they are almost invariably two-dimensional and written/read top to bottom left to right.

Before software, poetry was the primary mathematical template for language…

Can we achieve hypercomputation through language alone? Can we reframe traditional ontological assumptions about identity (in logic), space, and time? What about economics and value?

Grammar, for instance, as expressed by a sentence (i.e. – when we say something like the logical form expressed by a sentence) is always expressed in two-dimensions (more specifically, a two-dimensional plane). We might forget at times that syntax is therefore geometric.

Can we find examples where meaning in natural language is abruptly altered by the intercession of higher dimensions? Yes, readily. Consider street-signs that stand at a crossing, the abrupt cut-off of a sentence as it wraps around a building (say graffiti), and so on. *So, this should come as no surprise. It is obvious.* But there are other less obvious indicators of this.

Consider basic cryptographic techniques that literally encode one message into another, embedding one dimension of meaning into the second (and both being represented using 2 or more dimensions in an image say). <u>Cryptography is an interesting idea that hasn't been explored much in the philosophy of language</u> (though related notions like *implicature* and *impliciture* have). When we think about encryption we say at once the plentitude of meanings that arise in terms of intent, the prima facie meaning, and so on. Such messages often have additional value or purpose since they are used to relay important military commands. <u>These all arise at the intersection of many spatial dimensions.</u>

We will likely have to concede certain components of that more ambitious objective (i.e. - obtaining some new kind of ontological category but keeping *lines* as intrinsic to *symbols*; presupposing *space* without obtaining non-spatial conception of *symbols*, ...).

Now consider the following well-formed example:

No corresponding sentence - It's relational (it has relations) but it exceeds our conception of relations.

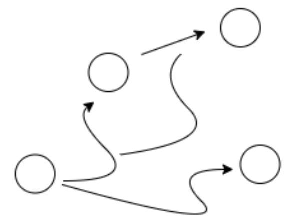


Fig. – Syntactically valid.

No corresponding sentence - It's relational (it has relations) but it exceeds our conception of relations.

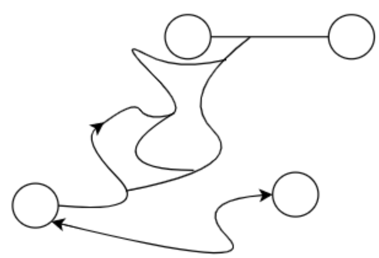


Fig. – Syntactically valid.

We observe that the figure immediately above illustrates something closer to a World State (state of affairs) with several "relational" connections existing even among the directed relational entities. One cannot extricate a subset of the depiction without losing something. (With a traditional *atomic ontology* like that offered by Russell, you can – that's the very point is that the basic facts of the world are separate though they might be incidentally or analytically *related* to each other).

Counter Argument / Reply: Connection Theory collapses into Graph Theory.

The informal depictions of graphs are justified and expanded (undirected, directed) here. Graphs could be depicted using any number of contrivances and is (actually) identified with the set-theoretic formulation conveniently depicted by and assisted by familiar circles and lines.

Graph Theory is probably the next-nearest cousin or sibling of Connection Theory. *Directed connectors* and *undirected connectors* are superficially related to the visual depictions of edges (undirected and directed) but they differ in a few key ways: (1) edges exist between vertices, (2) edge depictions are derived from and not identified with edges themselves, (3) combiners define the 'box' symbol - they are not free-standing entities like edges.

However, Connection Theory is capable of exceeding the power of Graph Theory - it can speak about all Categories.[4]

Specifically, other mathematical languages and theories are defined by or isomorphic to sub-fragments of Connection Theory.

Counter Argument / Reply: The notation implies the existence of Ersatz objects.

The `□` symbol fails to be an object itself. All the properties of any `□` token specify or are identical. Logical connectives are treated the same way so this should come as no surprise. In fact, that's one of the driving motivations to separate the *T*, *F* truth-values from other propositional variables or constants. All sentences in Classical Logic map to single shared singly instanced Truth-Values.

Furthermore, the `□` symbol is itself optional. I will demonstrate three methods by which to axiomatize Connection Theory – one of which involves using *Thinking Notion*.

3. Aims

Generally, mathematical and philosophical research aims need not converge, and they certainly do not necessitate the other from a practical standpoint. However, many of the watershed or landmark works in the history of mathematics have occurred due to, at least in part, the undergirding work of many philosophers (along with their quibbles about justification, entities, and conceptual clarification). These watershed or landmark events have typically fueled

[4] I had a referee at a philosophy once argue that Categories are just Graphs (I think because both are represented diagrammatically). That's plainly emphatically false. A brief look at the axioms demonstrates why.

significant innovation that eventually comes to drive most applied commercial engineering (computers, calculators, CPU's, etc.).

Perhaps that should come as no surprise since both Pythagoras and Archytas were foremost mathematicians during their time and devoted their efforts to the unity of thought, philosophical inquiry, mathematics, and the application of mathematics to the world we inhabit. From them the edifice upon which most early work in mathematics was set into place – affixed from the celestial realm of the empyreal and abstract down into the tangible stuff of the world and the concrete. That's not unique to mathematics of course, philosophical considerations have spawned many of the great intellectual achievements in science, mathematics, logic, and engineering.

So, it is both in line with the general development of mathematics and appropriate here to state some of the incidental aims of this project (some of these aims are not unique to this project but the combination of all of them is).

1. ***Philosophical***: To supply an objectless ontology for sensibly discussing ontologies that are <u>purely relational</u> in nature – that is which lack talk of any objects. Crassly, it's not that "everything's related" it's the perspective that "there are only interconnections, no things." Whether that be read metaphysically or not (here it is not).

2. ***Current***: To capture and express known ontological concepts that are presently essential to mathematical activity: sets, ordinals, functions, mappings, categories…

3. ***Posthuman***: To lay the groundwork for a (posthuman[5]), noun-less, proto-language.

   Not a language that is without symbol (body language or gestures will suffice) but that which whose meanings are devoid of the Predicate Subject/Object schemata (the denial of which was asserted to be primary in logic by Bradley and is common in certain Buddhist lines of thinking)

[5] Beyond human in some sense.

and which lack the kind of referential assumptions that engender philosophical angst.

The answer to Wittgenstein might be that human language, so conceived, is powerless to reveal. But like Heidegger observed, technology reveals – and herein, the aim of a new technology of and in language.

4. ***Ontological***: To rigorously define the conceptual equivalent of a 'set' to Set Theory or a 'category' to Category Theory. To define a fundamental "building block" (as it were) of a new way of viewing existing and novel mathematical theories.

5. ***Exploratory***: To lay out a system of axioms which may be selected, rejected, or expanded to see what results. Are there new kinds of mathematical entities that are free of the constraints of Set-Theoretic Foundations (particularly functions)?

6. ***Agnostic***: It is not eliminative – this is not an attempt to dislodge other systems of thinking but to supplement them. I take a *methodologically* non-foundationalist and broadly pluralist approach within mathematics (I proclaim no metaphysical allegiances – operating neither as a *Logicist* nor a *Formalist*. No claim to *Platonistism*, etc.).

7. ***Clarificatory***: To illuminate the implicit and largely abstracted[6] assumptions that guide functional analysis throughout mathematics today. We talk about functions as if one were to talk about decimal numbers but only by counting in groups of 10 or say constantly rounding to the nearest integer. There is a degree of obfuscating approximation that upon illumination may pioneer some great new work.

8. ***Univalent Foundations***: To link results here to emerging results in Univalent Foundations. Particularly with respect to *invariance*, *isomorphism*, *equivalence*, and *identity*.

9. To approach a fuller and more complete understanding about how <u>diagramming</u> (itself) works for any theory it can recover. Along the way,

[6] In the sense of a computer scientist.

to depict more clearly several representation theorems. To rigorously define a universal mechanism to unite diagramming in multiple areas (that are mostly done informally) but that are also nearly indispensable in their areas of inquiry.

10. ***(Potentially) Negative:*** If the findings here are partially or fully incorrect (or wrong in some way), it's useful to understand why. The arguments I give seem compelling to me, enough to warrant my attempting this effort. Illuminating what's wrong here might be even more valuable than a positive assertion about what's new or right.

4. Axioms and Formulation

This section introduces the core entities constituting the theory.

Definition 1. *Axiom Systems.*

Two ways at least:

i. Syntactic Approach – as a collection of sentences of some formal language.
ii. Combinatorial rules that conform to a generative grammar.

Those two approaches need not overlap though one can construct representation theorems, bridge laws, translation schemes, or intermediary languages between them (consider Venn Diagrams and Set Theory).

Note that the axioms of Connection Theory comprise a formal language and a ***Generative Grammar.***

Regarding axiom schemata - Consider:

(**AS1**) $A \to (B \to A)$

From ***Łukasiewicz's Simple Propositional Calculus.*** AS1 is expressed using non-sequent, non-tableaux, axiomatic calculus but is trivially expressed using the lambda equivalent.

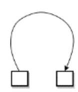
(**AS1**) $\lambda A \lambda B$: $A \rightarrow (B \rightarrow A)$

Here, A and B:

i. Are *uniformly bound* by the ***lambda operator***.
ii. Are *uniformly substituted into.*

Regarding the notation below. We may also express this using the familiar lambda calculus notation:

e.g. λ□: □|

e.g. λ□: 

However, here the `□` symbol (previously called a *leg* or *slot*) supports both ***non-uniform*** and ***always free substitution***. To formally define the profile of the `□` symbol:

i. *Optionally bound* by the ***lambda operator***.
ii. *Does not require uniform substitution (thereby).*
iii. *Is always freely substitutable wherever they are found in a well-formed formula.*
iv. *Play the role of variables in axiom schemata but with the above attributes.*

Connection Theory is also a diagrammatic theory (in the same vein as both Graph Theory and Category Theory). Diagrammatic construction to algorithmically construct the "building blocks" of various mathematical theories.

Connection Theory is closed (but not necessarily semantically closed per Tarski) in that its rules are expressed in Connection Theory (valid expressions of Connection Theory). Connection Theory is a metalanguage in which sub-fragments are object languages. It can construct these itself.

Connection Theory is a ***formalized***, ***axiomatic***, ***self-expressing***, ***closed***, ***generative grammar***.

5. *First Pass: Connectors and Connections*

A ***typed*** approach supporting the aims laid out in section 3.

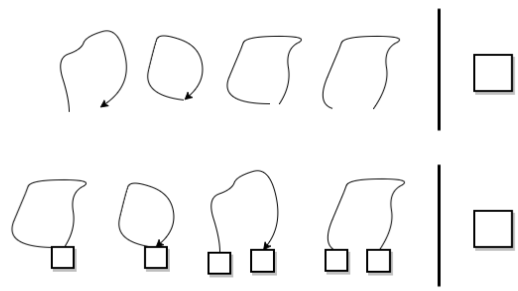
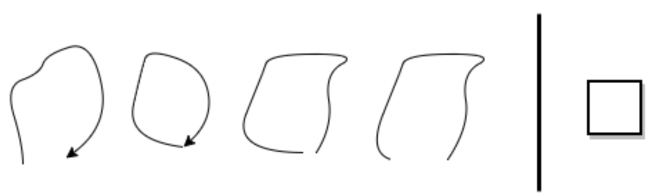


Fig. – All Connectors.

Definition. *Connectionless Connectors.*

First type. Connectionless Connectors have no connectors. They are only "attached" to other Connectors and never have Connectors "attached" to them.

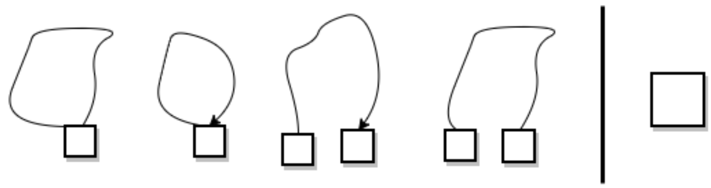
Four kinds: ***open directed***, ***closed directed***, ***closed undirected***, ***open undirected***.



Definition. *Connectors with connections.*

Second type. Connectors with connections. They are "attached" to other Connectors and have Connectors "attached" to them.

Four kinds: ***monadic undirected***, ***monadic directed***, ***dyadic directed***, ***dyadic undirected***.



Any Connector may be substituted into the `□` symbol

6. *Variant: Combiners and Connectors*

Definition 2. *Combiners*.

Previously a slightly different notation and terminology was used. These are equivalent.

Representation Theorem.

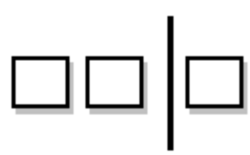
The main difference is that the former solely involved concatenations of diagrammatic images with no variation in procedural construction.

Below, we (TBD verify the use of I or We for this kind of math paper) simplify the process.

<u>Note that *iii* and iv (below) are and enable *recursive* combinatorial operations.</u>

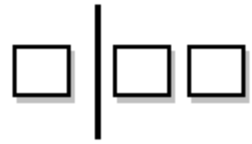
We shorthand and notate the procedural rules below noting that these are merely pictorial contrivances that can be used for simplicity.

i. Dyadic reduction.



*Natural Language Gloss: Removes a **combiner** where there were two.*

ii. ***Monadic introduction.***



Natural Language Gloss: Introduces a **combiner** *where there was one.*

iii. ***Monadic reduction.***

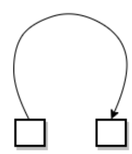
Natural Language Gloss: Removes a singular **combiner** *or links a* **combiner** *to a* **connector***.*

iv. ***Monadic reduction.***

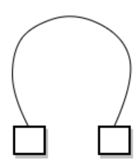
Natural Language Gloss: Introduces a singular **combiner** *or links a* **connector** *to a* **combiner***.*

Definition 2. *Connectors.*

i. ***Directed connector.***



ii. ***Undirected connector.***



Connectors – can represent functions, relations, morphisms.

A *theory of types (divvying up different manifestations)* for these kinds of relational entities is very interesting. Connectors are envisioned as top-level entities that represent, instantiate, or stand for those other entities (representationally, platonically, or schematically depending on the underlying assumptions one brings to the table).

Fig. Non-symmetry. "Directedness".

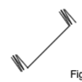
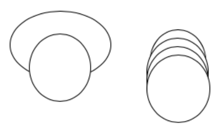


Fig. Equivalent syntax where the box operator is replaced with zig-zags



Fig, Compare these connectors - akin to functions or relations

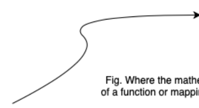


Fig. Where the mathematical concept of a function or mapping is presently at.

Fig. Equivalent syntax whereby the box operator is replaced with curves a connector.

7. Thinking Notation

Thinking Notation has been articulated elsewhere and attempts to lay a *descriptive* (the actual techniques or phenomenology of thinking) though *non-justificatory* (it does not offer proof-theoretic foundations) approach to the contents of thought. Thinking Notation is *axiomatic* and *generative*. It shares the same linguistic tokens as Connection Theory (symbols, lines, shapes)

As it turns out, *Thinking Notation* is sufficient to capture the semantics and operations of *Classical Sentential Logic*. Modifications to the original axioms provide full predication power sufficient for a theory of types and sets (though the specifics of ZFC set theory have not been worked out).

All of *Connection Theory*'s primary operations can be easily expressed in equivalent *Thinking Notation*.

Consider the following *thinking sequence*: # | #-# | #-# *-*| #-* | #-#

Begin* | *Replacement, Specificity* | *Existence* | *Specificity* | *Replacement

This is unexpected to say the least. There are two interesting take-aways here:

1. The development of symbolic logic (Boole, Peirce, Frege, Venn, etc.) accomplished two tasks – the formalization of logic and the creation language-systems sufficient to encompass all-known mathematical activity at that time.
2. *Thinking Notation* has some parallels to the above but greater flexibility. It can, for instance, capture zero and first-order notions (like Frege's system) but it can also express *Connection Theory*…

This is where things start to get much more interesting in terms of raw mathematical results…

8. Proofs and Operations

All proofs expressed in standard proof-theoretic semantics assume a representation theorem bridging a diagram to the natural language or proof-theoretic sequent expression. The use of names (variables are a heuristic)[7].

Fact. Given two-pronged Connectors A, B $\vdash$ three-pronged Connector AB.

A three-pronged connector is identified with two-pronged connectors under transformation (specified by the reduction or addition rules of the axiom system).

By mathematical induction, n-pronged Connectors …

This is interesting given that some of the mathematical objects that are constructed are novel. However, they are composed of fully legitimate entities.

[7]

Constructive *operations* are *closed* (involve only the basic buildings blocks of the theory). Below, *operations* also place the specific *combiner* rule used from one step to the next. The addition of the rule is only a convenience.

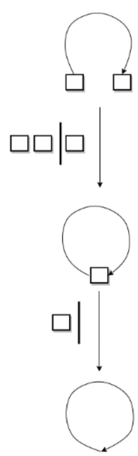


Fig. 1 – Constructing an identity morphism, automorphism, or self-reflexive relation.

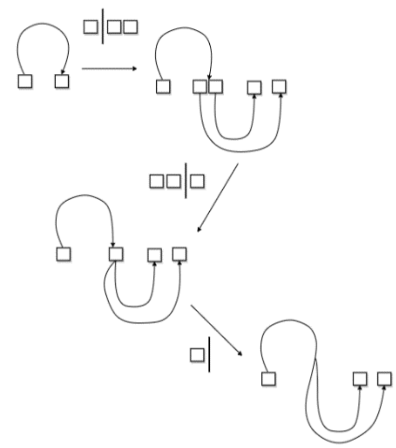


Fig. 2 - Constructing a one-to-many relation.

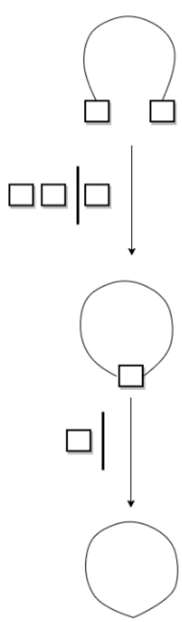


Fig. 3 – Constructing a point-free relation, an identity relation, and undirected automorphism.

Closure is a mathematical property of great importance to us here. *Closure* seems to be required to convert an otherwise arbitrary system of symbols into a *calculus*.

Mathematical systems are typically:

1. Closed under *deduction*, *implication*, etc.
2. The axioms are universal (for a specific domain) and thus, the theories are closed under those axioms, etc.

These well-known algebraic properties (studied in Category Theory) along with other familiar ones (*transitivity*, *commutativity*, etc.) are also captured in these target theories. Fragments of Connection Theory exhibiting these properties potentially divide useful fragments from less useful ones...

In the same way that sets are assumed as a background universe of entities in ZFC Set Theory (and then cherry-picked by the axioms of ZFC Set Theory so that any model of ZFC Set Theory is a model of the Ordinals and Natural Numbers), Connection Theories assumes a background universe of at least one Connector. And, sub-fragments of Connection Theory are adequate to model the Ordinals, Category Theory, etc.

Unlike, ZFC Set Theory, Connection Theory is explicit in terms of demonstrating the exact manner of these arrangements (e.g. functions are assumed, elemental inclusion operations are assumed, etc.). Furthermore, Connection Theory can probably be modelled as a "singleton monadic" theory.

9. Recovering Theories

The great aim of the original theorists who laid the foundations of mathematics during the late 19th and early 20th centuries was to begin with logic and end in sets. If their logic could recover (adequately model or represent) the Ordinals, the great task would be accomplished. Mathematics would be on consistent, rigorous footing (as it had always been envisioned). All cracks that had appeared in its exterior cleanly polished, filled, and removed.

Here, our ***semantics*** will be defined by the specific ***representation theorems*** that can be proven between our depictions and target theories.

To similar ends, we now set about recovering common mathematical objects, entities, and structures. Recoveries will be satisfied by a ***representation theorem*** (here, *isomorphism*). The proofs will be demonstrated diagrammatically.

Ordinals.

Definition.

Proof.

Categories.

Definition.

Proof.

Groups.

Definition.

Proof.

Sets. Sets can be represented in a variety of ways. Sets are often depicted using Venn diagrams, they can be represented *mereologically* (as kinds of topological entities in relationship), or in set-theoretic notation (mainly expressed through the addition of the elementary inclusion operator into First Order Logic).

We take this multiplicity of representational means to be the insight behind *representation theorems* – that there is some mathematical entity that can be represented or captured through a variety of target depictions.

Traditionally, the main aim of philosophers of mathematics and logic was to deliver Sets (and through extension, the Ordinals, Naturals, Reals, and therefore arithmetic calculations). Sets were seen as fundamental both in terms of status and in terms of a starting point. From then functions could be defined and then well-ordered sequences, and so on.

Here, we can construct an approach that reverse that sequence but nevertheless recovers sets and the ordinals.

Definition.

We can define a sub-fragment of Connection Theory using two Connectors constructed through the axioms specified previously.

One stands for a set, the other stands for an elementary inclusion operator (dyadic or binary function).

The dyadic function (arrow) is not *transitive*, it is not *reflexive*, and it is not *symmetric*. It can never be directed from one circle A to a circle B with an arrow pointed directly or indirectly at A. If we deny the previous constraint we end up with alternative (possibly naïve set theoretic) foundations.

> **Proof.** Obvious. Any expression of set theory can be expressed in pairwise set theoretic notation.... E.g. – '$A \in B$' or '$A \subset B$'. All expressions containing the proper subset symbol reduce to expression with the elementary inclusion operator. All set theoretic relations can be expressed with the *elementary inclusion* operator.

Proof. Obvious. Any expression of ZFC set theory can be expressed in pairwise set theoretic notation. Since no circle can be a member of itself (according to this sub-fragment of Connection Theory) so it is that all sets so proven are also well-formed in ZFC.

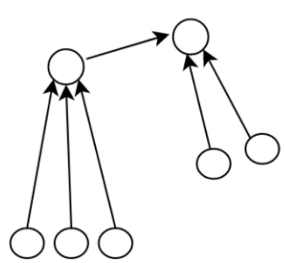


Fig. – Elementary inclusion depictions.

There are several equivalent sub-fragments. We can use Venn diagrams (and avoid constraints with using an arrow that led Cantorian naïve set theory into vicious circularities).

Venn diagrams are themselves captured in a sub-fragment of Connection Theory.

Feynman Diagrams.

Knot Theory.

10. Explaining Existing Phenomenon

Constructive proof. Connection Theory lends itself to Constructive methods without ruling on the Constructivist vs. Platonist Realism debate.

Existence proof. Every ontological item in use by mathematics today can be constructed from the axioms of Connection Theory (even Cantorian cardinalities[8]).

---

[8] It was proposed to me by the brilliant Dr. Han that *constructive finitude* in mathematics could be used to define traditional Cantorian cardinalities. One might introduce into a language L the concept of constructive finite (say, property *CF*) and then define the negation of *CF*. This proves problematic for several reasons but is an interesting idea primarily for those who are largely agnostic to the ontological status of these kinds of metaphysical questions.

As I see it, non-constructive mathematics explores Cantorian assumptions (which indeed seem to be required to do physical calculus in engineering and the sciences due to the use of the Reals, integrals, and limits).

On the other, there are difficulties with constructive approaches since if we can in fact introduce a concept of constructive finitude and then "just negate it" within any sufficiently rich language, we just end up with Cantorian math after all in any constructive mathematics.

Cantorian assumptions about cardinalities typically derive from certain considerations about one-to-one mappings, etc. This licenses the move from the Ordinals to the Reals (e.g. – there are more numbers ***between*** 0 and 1 than ***between*** 0, 1, 2, 3, 4, 5, …. Since 0/1, 0/2, 0/3, 0/4 … but also 1/3, 2/3, ¾, …). Either way, in every such system we'd get Cantorian AND constructive ordinals (constructive ordinals are a proper subset of Cantorian ordinals) and hence, we would prove the existence of a Cantorian predicate whether it had a model or not.

I have not formally addressed these questions…

Non-Constructive Hypothesis. For every constructive proof C for a theory or object T there is a corresponding non-constructive proof C^* for T.

Here, we will deploy the basic edifice of Connection Theory to demonstrate why, for example, two functions can be combined one after the other. Or, at the very least provide more insight into the opacity surrounding the combination of functions and their natures.

Generally, mathematical activity and mathematics has proceeded from (occasionally dogmatically held) axiomatic assumptions to increasing intelligibility about the exact procedures at play. Consider, for instance, Euclid *Elements* which upon closer scrutiny yielded several divergent geometric systems (the original axioms were held dogmatically as the true description of physical geometry).

Consider the following kinds of expressions (which can be harmlessly accepted) that find themselves wrapped up in several present axiom systems.

1. Given functions F, G there exists some function H such that $H = F \circ G$.

   Category Theory.

2. Given functions F, G such that:

   a. $F: \Phi \rightarrow \Delta$
   b. $G: \Phi \rightarrow \Omega$

   There exists some function H such that $H: \Psi \rightarrow \Phi$.

3. Given functions F, G such that:

   a. $F: \Phi \rightarrow \Delta$
   b. $G: \Omega \rightarrow \Delta$

   There exists some function H such that $H: \Delta \rightarrow \Psi$.

But the claim to existence is supported by assuming a background universe of sets (or structures), and some opaque notion about functions. Presumably this is justified in virtue of constructive claims. Either way, we may now explicitly demonstrate why this is possible:

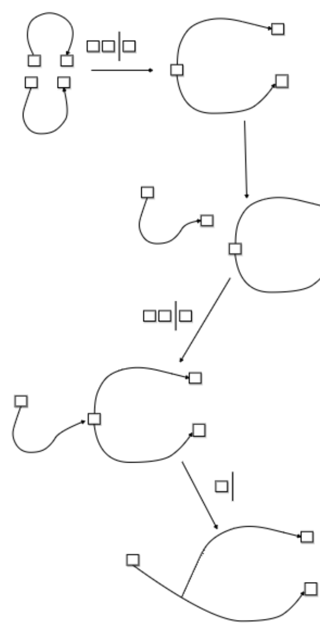


Fig. 4 – Step by step construction of a one-to-many relation.

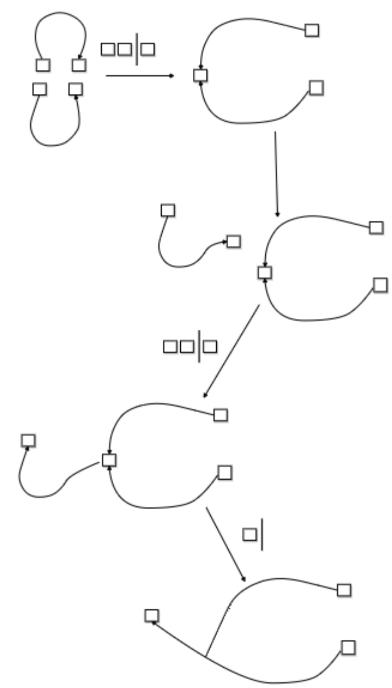


Fig. 5 – Step by step construction of a many-to-one function.

11. New Entities

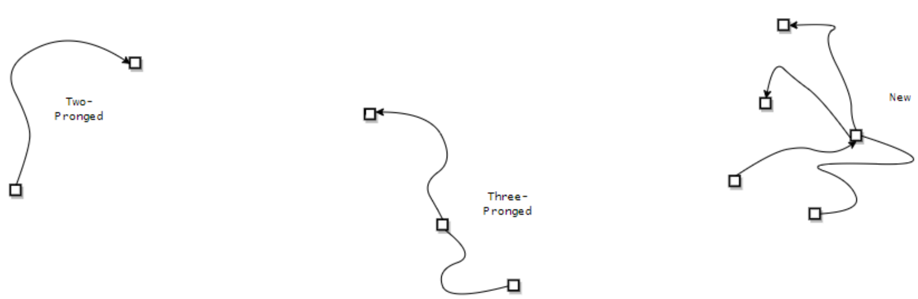


Fig. 6 – Valid syntax.

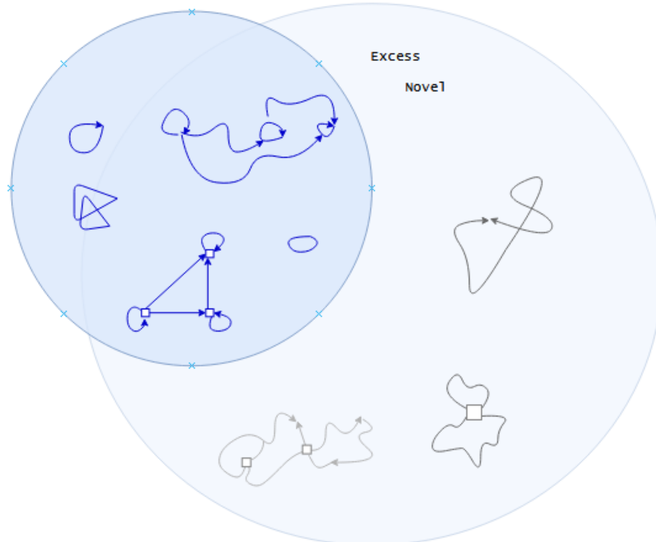


Fig. 7 – Petri Dish overlay of existing entities with novel ones (draft image).

Works Cited